Lec 26: 11/30/2010

Bounce and Shock Formation:

As we discussed last time, Iron nuclei survive all the way to nuclear densities. Also, neutrino trapping at densities higher than $\sim 10^{12}\, g\, cm^{-3}$ implies small lepton loss. As a result, the entropy increase during the collapse will be small and the core collapse will continue almost adiabatically.

The compression will be resisted by repulsive nuclear forces at short distances when the material reaches nuclear densities. This sends out a shock wave through the outer regions. The crucial question is how strong this shock will be after it has traversed the inner core. In particular, whether it will have sufficient strength to eject the stellar material in the form of an explosive event.

Such a complex issue requires detailed numerical simulations. The broad class of conditions under which the bounce and shock can occur can be bracketed by two limiting cases:

1) Complete neutrino trapping with minimum neutrino transport and a hard equation of state at transnuclear densities.

2) Neutrino transport decided by flux-limited diffusion and a relatively softer equation of state.

The first case will give a strong bounce shock, while the second one will lead to a more moderate initiation of the shock. The two cases will lead to similar evolutionary sequence until the epoch of core bounce. In $\sim 10^{-3}$ s after the trapping of neutrinos, the effects of the equation of state become evident and the evolutionary sequence differs. In both the cases a bounce shock does form and starts moving out through the core toward the outer region. At the point of bounce, the energy availabe to the shock is the kinetic energy, which for the case of complete neutrino trapping is $\sim 1.2 \times 10^{52}$ erg. As the shock moves, it heats the matter behind it. The heating leads to nuclear dissociation. Since the nuclear binding energy per nucleon is ~ 8 MeV, then $\sim 1.6 \times 10^{52}$ erg of energy per $M_\odot$ will be required for dissociation.

In most stellar models, the shock has to move through $\sim 0.9\, M_\odot$ of matter before it can even reach the Oxygen shell burning region. This requires approximately 1.44×10^{52} erg, which is still more than the available kinetic energy. Thus the shock barely reaches the region of Silicon burning covering $\sim 0.5\, M_\odot$ of material.

The above discussion shows that the numbers are evenly matched and small effects can change the conclusion in either direction. Therefore, the order of magnitude estimation is not reliable. However, even in the case most optimally biased toward an explosive shock the energy does not seem to be sufficient for an explosion.

The infalling matter into the stiff core will lead to an accretion shock in a manner similar to the case of protostar formation. This will add to the energy of the bounce shock. The rate at which mass is being fed into the accretion shock is:

$$\dot{M} \simeq 4\pi r^2 \rho c_s \qquad \left(c_s \propto \sqrt{\frac{P}{\rho}} \propto \rho^{\frac{1}{6}} \,,\ \rho \propto r^{-3} \right)$$

We therefore see that $\dot{M} \propto r^{-3/2}$, and hence the rate at which energy is fed to the bounce shock decreases as accretion continues.

The energy supplied by neutrino less from the core can help the explosion happen. Neutrinos that emerge from the neutrinosphere have a distribution function that is similar to that of the electrons from which they are produced. Neutrinos can heat the matter above the neutrinosphere through various interactions. The heating rate per unit volume scales $\propto \rho$. The cooling rate, however, will scale $\propto \rho^2$. Hence heating will dominate over cooling at low densities. If neutrinos can transfer energy to this region quickly, pressure can build up (due to rise in the temperature) and cause an explosion. This effect is also contributed to by possible convective transport in the outer regions.

One comment is in order at this point. Our discussion was based on some implicit assumptions, the most important of which are:

(1) The explosion is (nearly) spherically symmetric.

(2) The energy E involved in the explosion and the mass M that is ejected are such that $E < Mc^2$, i.e. the explosion is nonrelativistic.

(3) The major component of energy involved in the explosion is the kinetic energy of baryons that are ejected.

It is important to bear in mind that totally different cases are possible if the above assumptions are modified. For example, one extreme variant of a supernova explosion involves energy that is transported out almost as pure radiation. This is relevant for the phenomena of Gamma-Ray Bursts (GRB's).

Stellar Nucleosynthesis:

The most abundant elements in the universe are H and ^4He. Both of these elements are made at the very early stages of evolution of the universe, approximately the first three minutes after the Big-bang. This is done through Big-Bang Nucleosynthesis, which is in very good agreement with observations and is one of the observational pillars of the Big-Bang model.

The predicted mass fractions of H and ^4He are $\simeq 0.75$ and $\simeq 0.25$ respectively. However, these primordial values are affected by the stellar evolution at much later times. Nuclear burning in the stars results in a decrease in H abundance and increases ^4He abundance. For example, the abundance of H in the solar system is $\simeq 0.70$, while that of ^4He is $\simeq 0.28$.

Big-Bang Nucleosynthesis also makes tiny abundances of D,

^3He and ^7Li. These elements, however, are (almost) invariably destroyed in the stellar interiors.

All elements heavier than ^4He are made by the stellar evolution. Depending on the initial mass of the star, elements up to ^{56}Fe can be made during different phases of nuclear burning. However, elements significantly heavier than Iron cannot be made through these nuclear reactions because the binding energy per nucleon decreases for such elements.

There are other reactions that are capable of generating heavier elements, which for example take place in a supernova explosion.

One set of such reactions relies on the capture of neutrons by nuclei, which is not affected by Coulomb repulsion. Consider the absorption of a neutron by an element (Z, A):

$$(Z, A) + n \rightarrow (Z, A+1) + \gamma$$

If $(Z, A+1)$ nucleus is stable, then the neutron capture can continue to form $(Z, A+2)$, $(Z, A+3)$, ... elements. If it is unstable, on the other hand, it will undergo β-decay:

$$(Z, A+1) \rightarrow (Z+1, A+1) + \bar{\nu} + e^-$$

The crucial parameter that determines the different channels through which such reactions proceed is the neutron flux.

The mean time between the capture of two neutrons is given by:

$$\tau_{cap} = \left(n_n \langle \sigma_n v_n \rangle\right)^{-1}$$

Here n_n, v_n denote the number density and velocity of neutrons, respectively, and σ_n is the neutron absorption cross section. Depending on the β-decay lifetime τ_β two cases are possible:

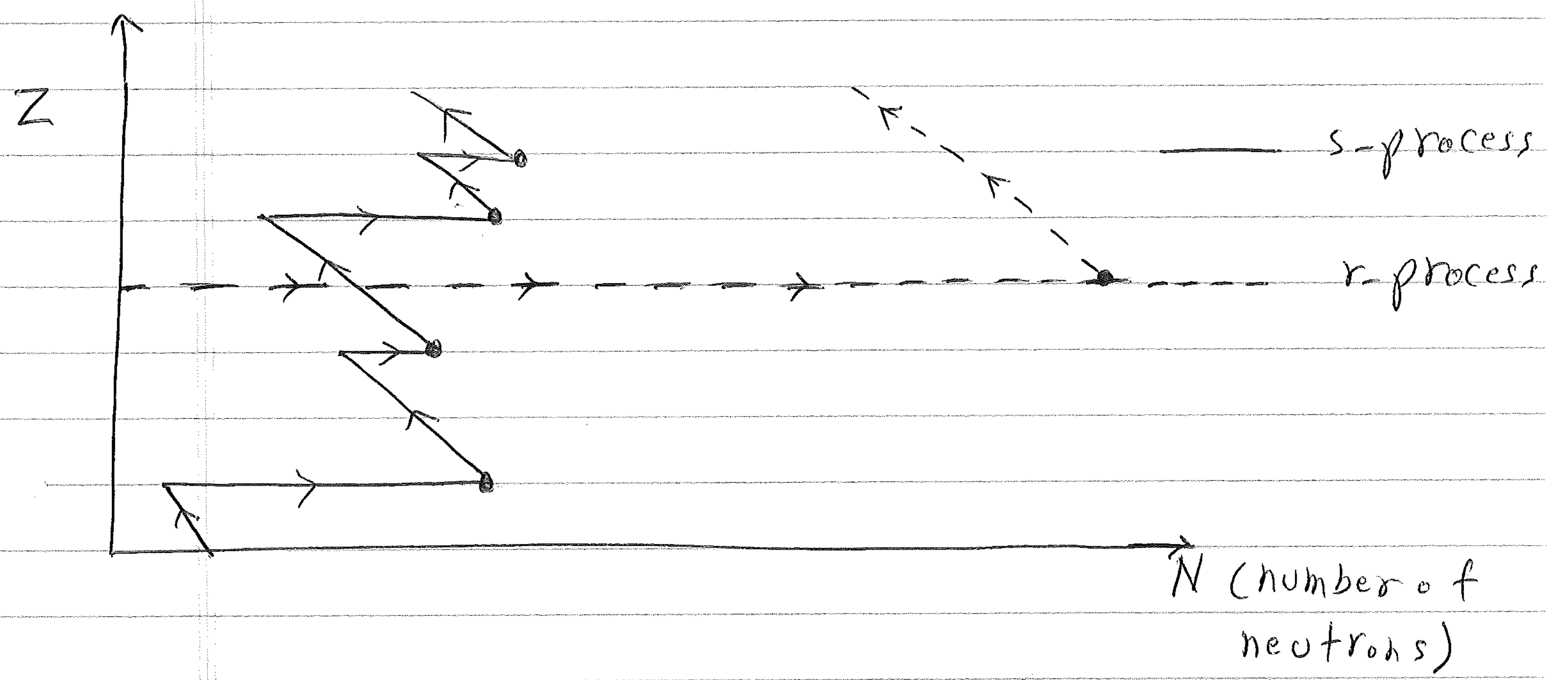
(1) If $\tau_{cap} < \tau_\beta$, then a further addition of neutrons is

very likely. This happens if the neutron flux is large. It leads to a sequence of r-process elements ("r" for "rapid neutron capture") that lie along the neutron rich side of stable elements.

(2) If $\tau_{cap} > \tau_\beta$, then β_- decay occurs before the second neutron is captured. This happens if the neutron flux is small. It leads to a sequence of s-process elements ("s" for "slow neutron capture") that lie along the continuous path of stable nuclei.

The τ_β generally ranges from a fraction of a second to a few years. For $\sigma_n \approx 10^{-25}\ cm^2$ and $v_n \approx 3\times10^8\ cm\ s^{-1}$, we have $\tau_{cap} \approx 10^8\ yr\ n_n^{-1}$. Thus for $n_n \lesssim 10^5\ cm^{-3}$, $\tau_{cap} \gtrsim 10^4\ s$ is larger than any β_- decay lifetime, leading to s-process. For $n_n \gtrsim 10^{23}\ cm^{-3}$, on the other hand, $\tau_{cap} \lesssim 10^{-6}\ s$, which is shorter than any β-decay lifetime, thus resulting in r-process.

The paths that s- and r-processes lie along are schematically shown in the following figure:



The horizontal lines correspond to neutron capture, while the diagonal lines correspond to β-decay. The dots • denote unstable nuclei.

It is important to point out the relevance of neutrinos for the r- and s-processes. As we discussed, the neutron flux is the determining parameter here. Neutron flux can be reduced through combining neutrons with electron-type neutrinos:

$n + \nu_e \rightarrow p + e^-$

The flux of ν_e thus affects neutron flux. The dynamics of neutrinos, including oscillation among the three flavors (electron-type, muon-type, tau-type) will be important in this regard.

Finally, we point out that r- and s-processes involving neutron absorption can also take place in red giants as a consequence of Helium flashes. During the peak burning phase, the temperature can be approximately 3×10^8 K. Then neutrons can be formed by means of reactions such as:

$^{22}Ne + \alpha \rightarrow {}^{25}Mg + n$

$^{13}C + \alpha \rightarrow {}^{16}N + n$

The large kinetic energies due to high temperatures can overcome the Coulomb barrier. Neutrons thus produced can have a significant flux and participate in r- and s- processes.